

# About the Free and Forced Nutation: The Daily Nutation

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**Abstract:** To explain the nutation phenomenon, Euler chose a geocentric frame of coordinates to present his dynamical equations. In accordance with his formulas, the nutation is caused by a momentum relative to the Earth's center, due to certain external forces. Further, Poinsot presented a special case of the Euler's equations, when any momentum of those certain external forces does not exist. Due to a problematic approximation, Poinsot announced that in this case may take place a nutation period of 10 month, named "free period", to distinguish it from the Euler's dynamical solution, known as "forced nutation". After Poinsot, the notion of "free nutation" was extended also to those nutation phenomena which have only a geophysical origin, without a momentum of certain external forces. Usually, the daily nutation is considered as being a free nutation phenomenon. In order to search for a forced daily nutation component, the daily trajectory of the Earth-Moon barycenter inside the Earth is used (a simple scheme of this barycenter trajectory is presented in a geocentric system of axes). Finally, if the ecliptic line is accepted to be described by the Earth-Moon barycenter, it must be accepted, too, that a torque due the Sun and the Moon acts during a sidereal day interval on diurnal Earth rotation around its axis. Due to the small value of the nutation constant, a period of 18,6 years is needed to correctly and completely detect the forced daily nutation; this phenomenon permanently presents very fine variations depending on the longitude of the ascending lunar node, the Moon's declination and the Sun's ecliptic longitude.

**Keywords:** Nutation, Euler-Poinsot Case, Earth-Moon Barycenter

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## 1. Introduction

In 1748, after two decades of astronomical observations, J. Bradley announced that the Earth's rotation axis presents some small perturbations from its multi-millennial precession way. J. Bradley also specified that the discovered phenomenon, named nutation, has a period of 18,6 years, similar to that one of the longitude of the ascending node of lunar orbit.

To explain the phenomenon of nutation period, L. Euler (1707-1783) considered the Earth as being an ellipsoid rigid body with a fixed point, and chose the origin of a fixed inertial reference coordinates system in order to coincide with the fixed point "O". The fundamental equation of the classical mechanics in this case, is

$$D\vec{K} / Dt = \vec{M}Fex \quad (1)$$

where  $\vec{K}$  is the kinetic moment of the body with a fixed

point,  $D\vec{K} / Dt$  is the differential of  $\vec{K}$  relative to the fixed inertial frame of coordinates,  $\vec{M}Fex$  is the moment of the resultant of external forces relative to the fixed point.

In a relative movement, when the mobile axes  $Oxyz$  are fixed in the body and this frame has the origin also in "O", the relation (1) becomes:

$$d\vec{K} / dt + \vec{\omega} \times \vec{K} = \vec{M}Fex \quad (2)$$

where  $\vec{\omega}$  represents the instantaneous rotation velocity which passes through the point "O", and  $d\vec{K} / dt$  is the differential of  $\vec{K}$  relative to the mobile frame.

In 1768, L. Euler also introduced the moment of inertia relative to a specified axis, a strict positive entity, which measures the body resistance to the torque relative to the same axis [1]. Thus, we have:

$$\begin{aligned} A\dot{p} + (C - B)qr &= Mx \\ B\dot{q} + (A - C)rp &= My \\ C\dot{r} + (B - A)pq &= Mz \end{aligned} \tag{3}$$

where  $A, B, C$  are the moments of inertia (the mobile axes coinciding thus with the main inertial axes),  $p, q, r$ , are the  $\vec{\omega}$  components in the mobile frame, and  $Mx, My, Mz$  are the  $\vec{M}Fex$  components in the same mobile frame. Naturally, the solid body being a rigid one,  $A, B, C$  have constant values (after some decades, J. Liouville extended the equations system (3) to mobile axes which do not coincide with the main axes of inertia).

The astronomical determination of the stars positions is made in the equatorial frame (ascension, declination); this is why the apparent stars position must be corrected with the so-called nutation coefficients. Must be noticed the great contribution of E. Woolard, who, in 1953, presented the 109 terms which defined the nutation coefficients. The arguments in his relations are: the Moon's longitude, the perigee, the nodal line, the Earth's ecliptic longitude and its perihelium position [2].

Sometimes, in precise computed programs related to the apparent position of the stars, the nutation coefficients are calculated in a matrix form, together with the precession values, in the subroutine PRENUT.

## 2. Particular Cases

We present below two particular cases of the movement of a rigid body with a fixed point, when all the external forces pass only through the fixed point and, subsequently, the moment of the external forces is zero ( $\vec{M}Fex = 0$ ).

For (1) and (2), we have:

$$D\vec{K} / Dt = 0 \tag{4}$$

$$d\vec{K} / dt + \vec{\omega} \times \vec{K} = 0 \tag{5}$$

### 1) First case

We assume for a moment that the vectors  $\vec{\omega}$  and  $\vec{K}$  are both situated on the body's main and principal axis of inertia; in this case, the vectorial product  $\vec{\omega} \times \vec{K} = 0$ . Then (5) becomes:

$$d\vec{K} / dt = 0 \tag{6}$$

In astronomy,  $D\vec{K} / Dt$  is implicate in the nutation phenomenon;  $d\vec{K} / dt$ , from the geodesical point of view, may indicate a latitude variation. In this case, (4) and (6) certify that, if the external forces pass only through the fixed point, no nutation exists, and also no latitude variation can take place. In the theoretical mechanic, this case is known as a stable position [3].

### 2) Second case

We assume for the moment that we have also, as in the above case,  $D\vec{K} / Dt = 0$ , and the vectors  $\vec{\omega}$  and  $\vec{K}$  are

situated on the same main and permanent axis of inertia. Next, (like in the gyroscope case) a non-expected external event takes place ( $\vec{M}Fex$  being suddenly not zero) but, after a limited time duration, this event stops (like in the gyroscope case, too). In this case, naturally, the inertia moments of the rigid body relative to the main axes of inertia remain unchanged, but the vector  $\vec{\omega}$  is no more positioned along the main permanent axis of rotation and  $\vec{M}Fex$  turns again to zero (here ends the resemblance with the gyroscope case). If the rigid body is considered to be a rotational ellipsoid (that means  $A = B$ ), the equations system (3) becomes:

$$\begin{aligned} A\dot{p} + (C - A)qr &= 0 \\ A\dot{q} + (A - C)pq &= 0 \\ C\dot{r} &= 0 \end{aligned} \tag{7}$$

So, L. Poinsot presented in 1834 the Euler-Poinsot case, the  $\vec{\omega}$  vector no more coinciding in direction with the main and permanent axis of inertia, and  $\vec{M}Fex$  being zero [4].<sup>1</sup>

A very known approximative solution in this case suggests a variation of latitude for 10 months, duration named Euler's period. Since no external impulse is supposed to exist ( $\vec{M}Fex = 0$ ), usually the Euler-period case is called also free nutation, in order to distinguish it from a forced nutation, when a real moment of external forces acting on fixed point exists, disturbing the rotation axis position from its precession way.

After some years of geodesical determinations made around 1900, S. C. Chandler announced the detection of the first periodical latitude variation, which lasts for 14 months instead of 10 months (as it was expected) [6].

In the same time, also around the beginning of the XX<sup>th</sup> century, the existence of the Earth tides was recognized as being a real one, and the internal structure of the Earth began to present its complexity [7]. According to the Newcomb's suggestion, the Earth must be considered, in the Euler-Poinsot case, no more being a rigid body, but being a deformable one.

During the XX<sup>th</sup> century, many studies, without success, tried to theoretically explain the four months difference between the Euler's period and the Chandler's observed period in latitude variation (named also "Chandler period"). Solving this problem presents great difficulties.

Besides the equations (7), the formulas related to the problems of deformable bodies must be taken account, too; this implies some conditions in order to respect the continuity equation, and conditions related to the fixed point, which mustn't be subject of any displacement (boundary condition). We made in 1978 some simple attempts to solve these problems by using the Lagrange variable (used in space problems of deformable body [8]), and we found that the

<sup>1</sup> The Euler-Poinsot case, in our opinion, may be also regarded like an unstable case as part of the stability theory (this theory appeared some decades after L. Poinsot) [5].

simple model of the Earth's elasticity cannot justify an increasing length of the Euler's period (10 month) in order to reach the Chandler's period (14 months). We published this result in 1991 [9].

Only after some decades (in 2019), we decided to publish the rigorous solution of the Euler-Poinsot case, where we have shown that the EULER PERIOD DOES NOT EXIST in the rigid model case of the Earth with a fixed point [10].

Must be noticed that during the XX<sup>th</sup> century, studies regarding the Earth's rotation around its axis have experienced a very great development, very important international institutions being created (BIH, ILS, IPMS, IERS [11]). Many precise and important results regarding the Earth's rotation around its axis were obtained and a secular component was discovered.

### 3. The Daily Nutation

The possibility that the Earth rotation axis may also exhibit a daily nutation, was theoretically proposed by T. Sloudsky and S. S. Hought in 1896 [12].

In April 1961, during an international conference of the commission of the Earth's rotation, N. N. Pariiskii made a proposal to the assembled astrometrists to attempt to detect a real diurnal nutation from long series of astronomical observations of the latitudes. The first one to achieve this goal successfully was N. A. Popov in 1963, using 23 years of latitude observations made at the Poltava Gravimetric Observatory; he discovered and announced a diurnal nutation of amplitude  $a=0,016\pm 0,004$  (later, N. A. Popov reported an improved value of  $0,020$ ) [12].

It is very important to mention the remarkable results of C. Sugawa regarding the daily nutation. In 1965 he announced a very clear relation between the Moon's hour angle and the latitude variation detected in residual latitudes at Mizusawa. C. Sugawa used the residual latitudes obtained with the visual zenith telescope during 1935-1961, and with the floating zenith telescope during 1955-1961. The above results were considered as being caused by the lunar tidal effect on the vertical [13].

A remarkable work, published in 1968 by S. Debarbat, was related to some periodic terms which appear in the residuals, in time and latitude of the observations made at the Paris Observatory between 1956 and 1963, with the OPL model of the Danjon's astrolabe [14].

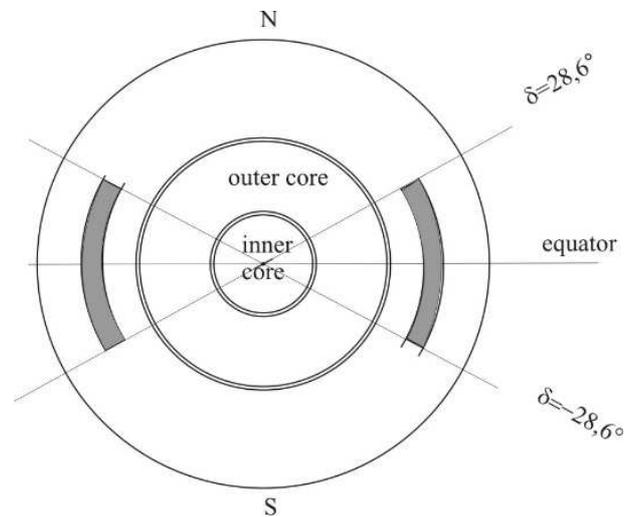
As a consequence of the approximative solution in the Euler-Poinsot case, the daily nutation was considered as being only due to a free nutation (it was no acting moment of the external forces, and the Earth was regarded as a non-rigid body).

Due to the small value of the nutation constant ( $N=9,2$ ), it is difficult from the astronomical point of view to detect a forced diurnal nutation during a diurnal sidereal period (special instruments and long interval of observation time interval - at least 18,6 years - are needed). This is why the results obtained by N. A. Popov and C. Sugawa are very important, because they prove the existence of a real diurnal nutation which has implication on the latitude variation.

Naturally, a question arises: is the diurnal nutation caused only by a free nutation?

If a forced daily nutation exists, it must be firstly related to a diurnal period phenomenon, like the diurnal Earth rotation around his axis.

Frequently, the ecliptic line is mentioned as being the Earth-Moon barycenter trajectory.



**Figure 1.** The Earth–Moon barycenter belt (dark zone), a section inside the Earth through the Earth's rotation axis.

This is why we used firstly some simple attempts regarding the limits of the Earth-Moon barycenter inside the Earth, and presented them in a recent paper [15]. We firstly considered that the Earth and its geological composition have a homogeneous and spherical structure. As the distance between the Earth and the Moon varies between the apogee and the perigee, it is obvious that the barycenter may be situated only inside two concentric spheres (with the radii of 4.335 km and 4.941 km, respectively), having their center in the same Earth center. Due to the fact that the Moon's declinations can be situated only between  $28,6^\circ$  north and  $28,6^\circ$  south of the celestial equatorial plane, the barycenter locus is putatively located only between the terrestrial latitudes  $20,55^\circ$  north and  $20,55^\circ$  south, inside the two concentric spheres mentioned above (Figure 1), like a torus shape around the rotation axis.

Further, we briefly present the barycenter's trajectory during a diurnal period and a nutation period.

#### *The diurnal period*

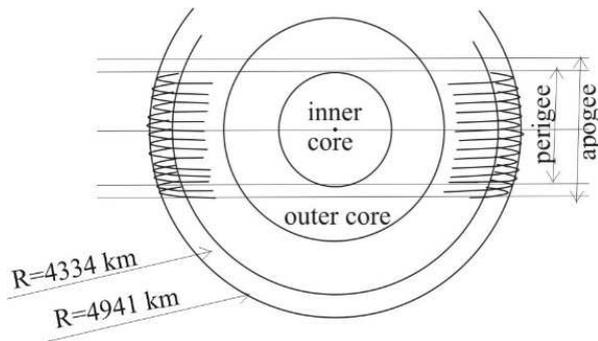
Due to the fact that, in a sidereal day, the Earth rotates  $360^\circ$  around its axis and the Moon walks only at most  $13^\circ$  on its orbit way, the barycenter has a diurnal retrograde motion inside the Earth, as seen from somebody situated at the Earth's north pole. The barycenter's diurnal trajectory can be assimilated with an approximative circular spiral, which turns around the Earth's rotational axis in a helicoidal form during a draconic period.

#### *The nutation period*

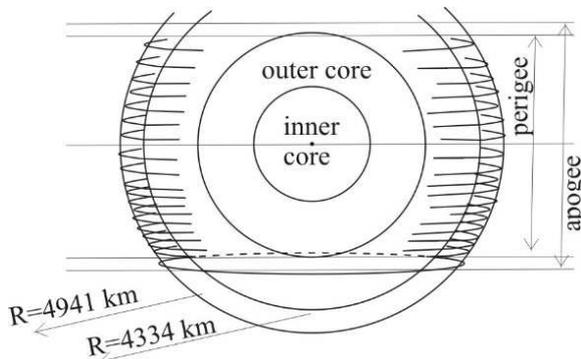
During the 18.6 years nutation period, there are two extreme positions:

When the Moon's declinations vary between  $+18,3^\circ$  and  $-18,3^\circ$ , all the 27 daily spirals surround only the inner Earth's core, which has a radius of 1220 km [7]; in this case, the ecliptic longitude of the ascending node of the lunar orbit line is  $180^\circ$  (Figure 2).

When the Moon's declinations interval is situated between  $+28,6^\circ$  and  $-28,6^\circ$ , the barycenter spiral surrounds, in a more spreading form, the entire Earth nucleus, which has a radius of 3500 km [15]; in this case, all the 27 daily barycenter spirals are more distant from each other when crossing the same meridian, than in the previous case; the ecliptic longitude of the ascending node of the lunar orbit is  $0^\circ$  (Figure 3).



**Figure 2.** The spiral described by the barycenter trajectory when the longitude of the mean ascending node is  $180^\circ$ .



**Figure 3.** The spiral described by the barycenter trajectory when the longitude of the mean ascending node is  $0^\circ$ .

It is obvious that, due to the Moon's trajectory, the barycenter is permanently changing its position inside the Earth. That means that the daily spirals dimensions and positions related to the Earth's equator, vary continuously, depending on the Moon's declination.

## 4. Conclusion

If we accept that the ecliptic line is described by the Earth-Moon barycenter, we must also accept that a torque, due to the Sun and relative to the Earth's center, may exist, which may cause a forced daily nutation.

The daily variation of the latitude may be caused not only by a free nutation due to a geophysical origin, but also by a forced nutation due to the Sun's action on the Earth-Moon barycenter.

To detect a daily nutation, it needs at least 18,6 years of astronomical observation. If a forced daily nutation exists, its value depends on the Moon's declination and on the Sun's ecliptic longitude.

We stop here. Our aim was to present only some points of view regarding the forced and the free nutation.

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